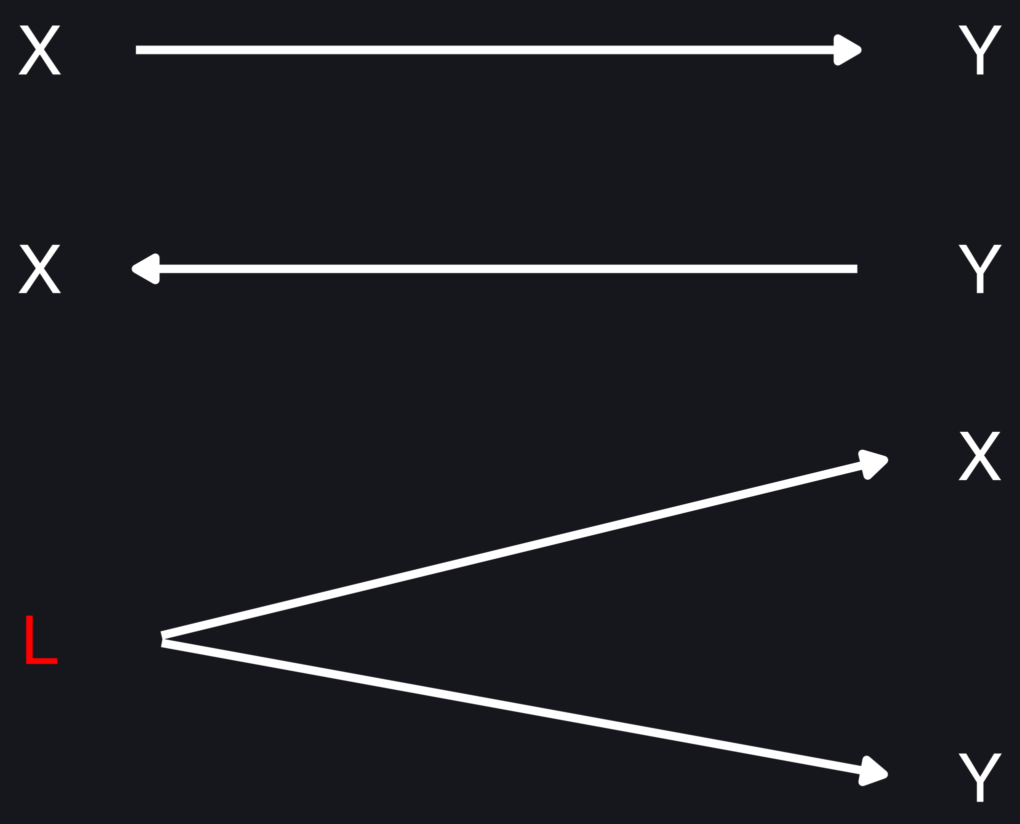
Introduction to PAGs + FCI + TFCI

June 4, 2024

No unmeasured confounders?

DAGs (and CPDAGs) assume no unmeasured/latent confounders (causal sufficiency). Often a strong assumption.



All 3 cases: $X \not\perp\!\!\!\perp Y$.

Same conditional independencies (of observed variables)

Need something more to represent/visualize relationships

Latent/unmeasured confounders

If we can "imagine" latent confounders we can still use DAGs for causal inference and determine what we can estimate.

How to visualize the conditional independencies among the **observed variables only** for DAGs with *observed* and *latent variables*?

We could marginalize out all latent variables.

DAGs are not closed under marginalization so cannot be represented by a DAG

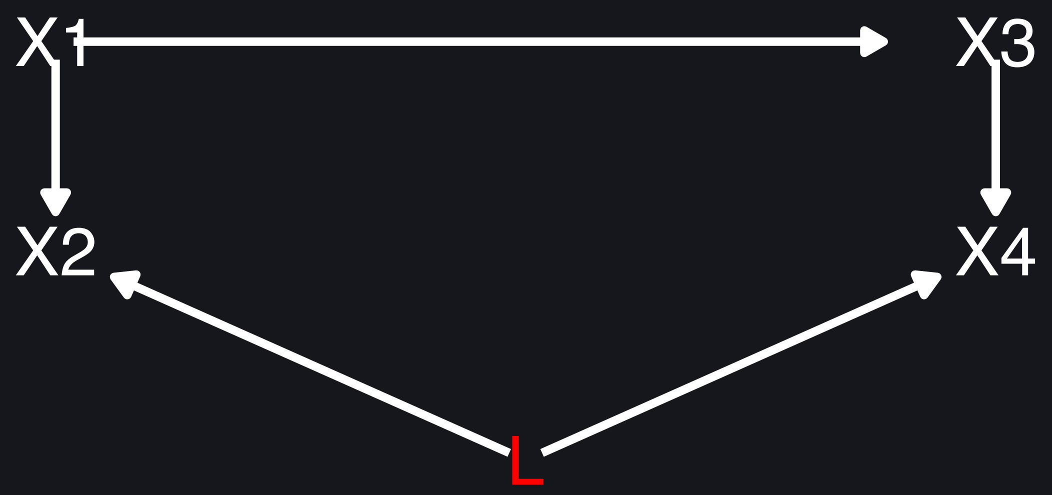
Maximal Ancestral Graphs (MAGs)

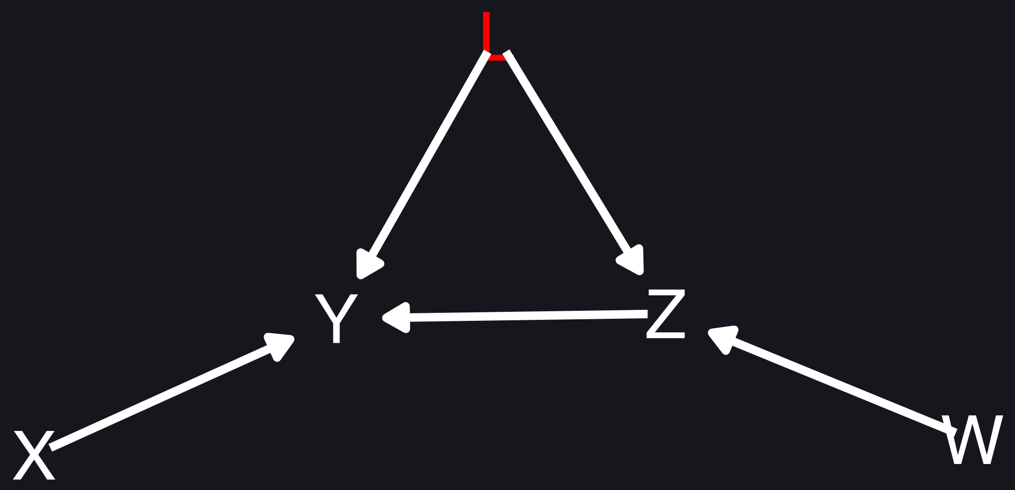
A *maximal ancestral graph* is a (directed) *mixed ancestral graph* that

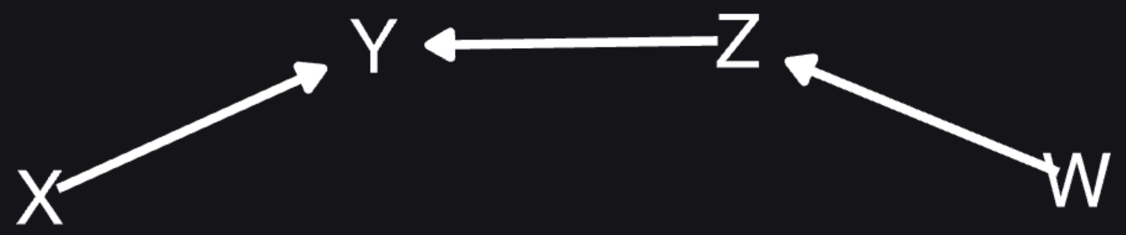
- may contain two kinds of edges:
  directed edges (→) and bi-directed edges (↔).
- has no directed cycles
- an edge X ↔ Y means *no* directed path from X to Y, or Y to X.
- is maximal: no edge can be added without changing the independence model.

For DAGs we have d-separation. For MAGs we have m-separation.

Examples







Partial Ancestral Graphs (PAGs)

Causal discovery can only hope to find a PAG = an equivalence classes of MAGs. Not *the* MAG.

Note: causal *ancestors* - **not** direct causes.

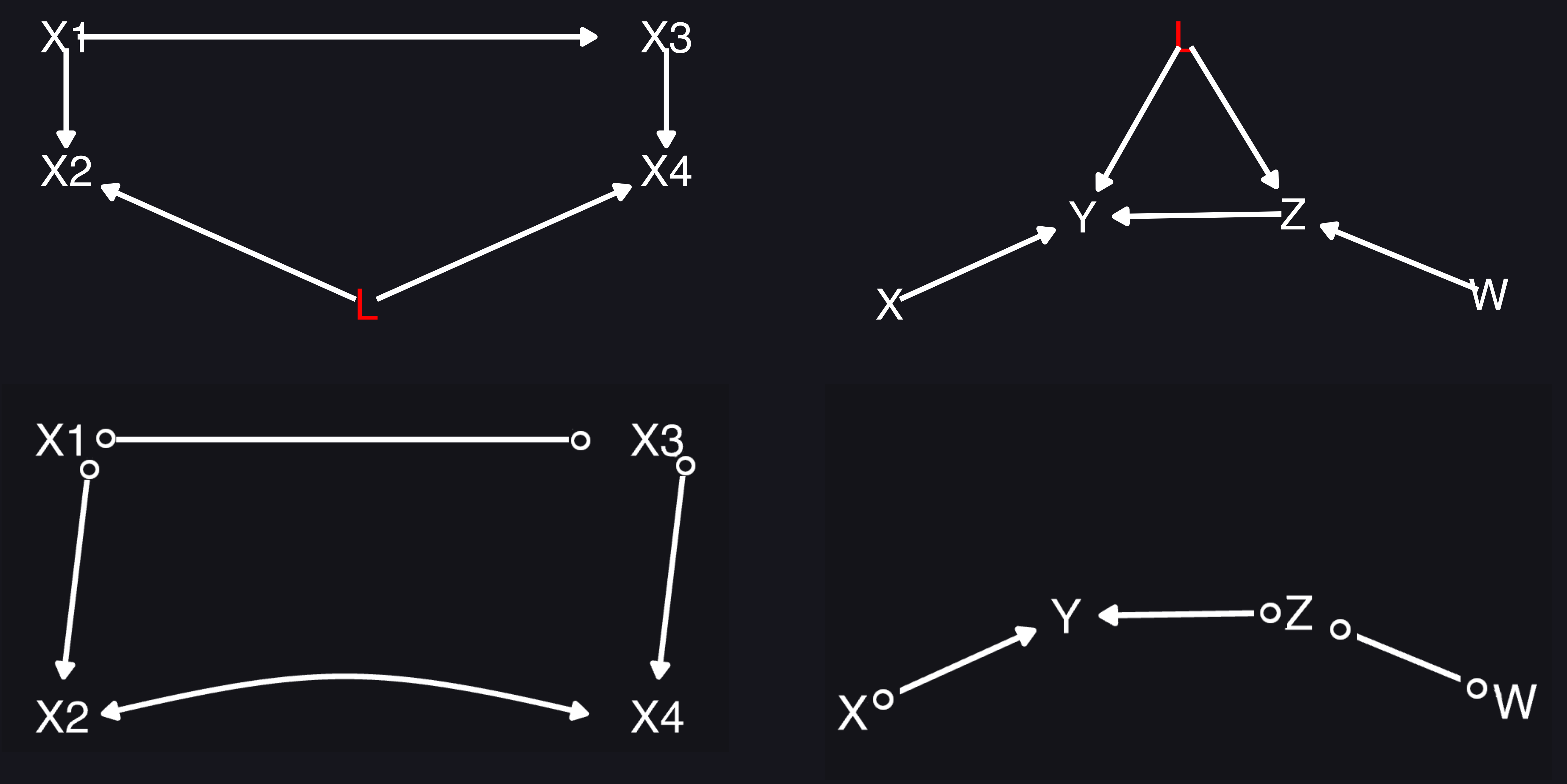
Three edgemarks in PAGs have the following interpretation:

- Blank: this *blank* is present in all MAGs in the equivalence class.
- Arrow: this *arrow* is present in all MAGs in the equivalence class.
- Circle: at least one MAG in the equivalence class has the edgemark as *blank*, and at least one has the edgemark as *arrow*.

Edge interpretation in PAGs

Edge	**Meaning**
Directed $X \rightarrow Y$	X is an *ancestor* of Y, and there *may* further be unobserved confounding between the two
Bidirected $X \longleftrightarrow Y$	*unobs confounding* between X and Y, but *no* causal ancestral relationship in either direction
Possibly bidirected edge $X \circ\!\!\rightarrow Y$	either $X \rightarrow Y$ or $X \longleftrightarrow Y$
Undetermined edge $X \circ\!\!-\!\!\circ Y$	no info about the relationship between X and Y. Either $X \rightarrow Y$, $X \leftarrow Y$ or $X \leftrightarrow Y$.

Examples



Fast Causal Inference (FCI)

0. Start with fully connected graph
1. Learn initial skeleton using tests of conditional independence **from neighbours**. May contain too many edges due to sep-sets
2. Find v-structures ($C \to A \leftarrow B$)
3. Determine larger separation sets & test for conditional independence. Get new skeleton and v-structures.
4. Orient remaining edges using ruleset

Result: a PAG

The temporal FCI algorithm (TFCI)

Extends the FCI algorithm to tiered information. Mimics TPC

1. For conditional independence test between X and Y do not condition on the future: No variables from a later tier than both X and Y in the separating sets.
2. Same modification here. No variables from a later tier than both X and Y in the separating sets.
3. Orient remaining edges using ruleset. Start by setting arrowheads at latest node for edges spanning tiers.

References

P. Spirtes, C. Glymour, and R. Scheines. *Causation, Prediction, and Search*. The MIT Press, Cambridge, Massachusetts, 2nd ed., 2000.

P. Spirtes, C. Meek, and T. Richardson. *An algorithm for causal inference in the presence of latent variables and selection bias.* In Computation, Causation, and Discovery. 1999.

J. Zhang. *On the completeness of orientation rules for causal discovery in the presence of latent confounders and selection bias.* Artificial Intelligence, 172(16-17), 2008.

R Ali, T. Richardson, and P. Spirtes. *Markov equivalence for ancestral graphs*. The Annals of Statistics, 37(5B):2808–2837, 2009.